Pythagorean Triangular and Trapezoidal fuzzy assignment problem-Formulations and its Solution

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Abstract— The assignment problem is a recognized optimization challenge. Incorporating uncertainties in the problem parameters complicates the efficient resolution of the assignment problem, a frequent occurrence in real-world situations. Pythagorean fuzzy uncertain variables provide a more thorough and balanced depiction of uncertainty by integrating the ideas of uncertainty theory with the advantages of fuzzy logic. Triangular and trapezoidal fuzzy numbers are employed in this paper. Additionally, it employs the branch and bound algorithm to address the Pythagorean ambiguous uncertainty assignment problem. A numerical exemplification is employed to illustrate the algorithm's intricate steps. A novel ranking method is suggested for triangular and trapezoidal fuzzy numbers. The feasibility of integrating Pythagorean fuzzy uncertainty into optimization problems is underscored by the findings.

Index Terms— Assignment Problem, Fuzzy numbers, Pythagorean fuzzy number, Triangular Pythagorean fuzzy numbers, Trapezoidal Pythagorean fuzzy numbers, Branch and Bound method.

I. Introduction

In the workplace and other contexts, assignment problems are crucial. In an assignment problem, "n" jobs are to be completed by "n" people, based on how well they can complete the work. We suppose that each individual can perform a maximum of one work and that each person can be assigned exactly one job. Finding the best assignment to do all tasks at the lowest possible cost or with the most possible profit is the primary goal of assignment problems. Optimizing assignment cost is applied in various fields such as logistics and transportation, workforce scheduling, resource allocation, project assignment, matching algorithm and sports scheduling. It is a resource that assists project managers in the efficient planning, scheduling, and management of initiatives. When the parameters are ambiguous, fuzzy concepts are employed to resolve the issue. The notion of fuzzy sets (FSs), referred to as type-1 fuzzy sets, was proposed by Zadeh [1]. These sets employ membership functions to characterize uncertainties. Their success across several sectors can be ascribed to their ability to manage ambiguity. Numerous researchers have proposed various extensions of fuzzy sets, including picture fuzzy sets [2], q-rung Ortho pair fuzzy sets [3], type-2 fuzzy sets [4], interval type-2 fuzzy sets [5], intuitionistic fuzzy sets [6], neutrosophic sets [7], hesitant fuzzy sets [8], and Pythagorean fuzzy sets [9], among others. As FSs can solely convey vagueness, they are incapable of addressing the indecision inherent in human cognition. To facilitate a clearer articulation of the hesitations, Atanassov [6] introduced intuitionistic fuzzy sets (IFSs), a significant extension of fuzzy sets. This approach uses the degree of membership and non-membership to signify imprecision and ambiguity; the total of these two membership degrees must not exceed 1. The primary advantage of IFSs is their capacity to mitigate uncertainty arising from inadequate information. Nevertheless, if the aggregate of (membership) + (non-membership) ≥ 1, the IFSs are unable to function effectively. Pythagorean fuzzy sets (PFSs) were introduced to address this intuitionistic fuzzy set (IFS) issue. Yager and Abbasov [8] developed PFSs to enhance the representation of IFSs through the degrees of membership and non-membership. Pythagorean fuzzy sets (PFSs) are advantageous extensions of intuitionistic fuzzy sets (IFSs) that offer an innovative approach to addressing ambiguity in membership degrees. The aggregate of the two degrees may exceed or fall short of one, despite the total of the squares of the two degrees being less than or equal to one. Probabilistic fuzzy systems (PFSs) excel at handling ambiguity and imprecision inherent in human cognition and subjective assessments [10]. Our analysis indicates that, in contrast to IFSs, PFSs offer greater flexibility and capacity to articulate uncertainty due to the broader range of membership degrees in PFSs compared to IFSs [11]. As PFSs are extensions of IFSs, they inherently pertain to the metric space of the IFSs. In addition to the benefits of IFSs, PFSs offer an expanded search space to encompass agreement, disagreement, and hesitation in decision-making [11]. In multiple facets, When the parameters are unclear, ambiguous notions are utilized to address the problem. The concept of fuzzy sets (FSs), known as type-1 fuzzy sets, was introduced by Zadeh [1]. These sets utilize membership functions to define uncertainties. Their success in various fields can be attributed to their capacity to navigate uncertainty. A multitude of researchers have suggested diverse extensions of fuzzy sets, encompassing picture fuzzy sets [2], q-rung Ortho pair fuzzy sets [3], type-2 fuzzy sets [4], interval type-2 fuzzy sets [5], intuitionistic fuzzy sets [6], neutrosophic sets [7], hesitant fuzzy sets [8], and Pythagorean fuzzy sets [9], among others. Since fuzzy sets can only express ambiguity, they are unable to resolve the uncertainty intrinsic to human thought. To elucidate the uncertainties, Atanassov [6] created intuitionistic fuzzy sets (IFSs), a notable expansion of fuzzy sets. This method employs the degrees of membership and non-membership to denote imprecision and ambiguity; the sum of these two membership degrees must not surpass 1. The principal benefit of IFSs is their ability to alleviate uncertainty stemming from insufficient information. However, if the sum of (membership) and (non-membership) is more than or equal to 1, the IFSs cannot operate efficiently. Pythagorean fuzzy sets (PFSs) were developed to resolve the challenges associated with intuitionistic fuzzy sets (IFS). Yager and Abbasov [8] established PFSs to improve the depiction of IFSs via the levels of membership and non-membership. Probabilistic fuzzy sets (PFSs) are beneficial extensions of intuitionistic fuzzy sets (IFSs) that provide a novel method for managing uncertainty in membership degrees. The sum of the two degrees may surpass or be less than one, even when the total of the squares of the two degrees is less than or equal to one. Probabilistic fuzzy systems (PFSs) are adept at managing the ambiguity and imprecision intrinsic to human cognition and subjective evaluations [10]. Our analysis reveals that, unlike Intuitionistic Fuzzy Sets (IFSs), Pythagorean Fuzzy Sets (PFSs) provide more flexibility and the ability to express uncertainty, owing to the wider spectrum of membership degrees in PFSs relative to IFSs [11]. Since PFSs are extensions of IFSs, they automatically relate to the metric space of IFSs. Besides the advantages of IFSs, PFSs provide an augmented search area that includes agreement, disagreement, and hesitation in the decision-making process [11]. Pythagorean fuzzy numbers (PFNs) surpass intuitionistic fuzzy numbers (IFNs) and fuzzy numbers (FNs) in some aspects. PFNs facilitate the modeling and alleviation of uncertainty's impact in fuzzy logic systems. Luqman et al. [16] characterized triangular probability fuzzy numbers and employed them for risk evaluations. LR-type PFNs were delineated by Akram et al. [17] and employed to address PF linear programming challenges. A robust theory for generalized PFNs was recently formulated by Akram et al. [18,19] and Habib et al. [20], and applied to the hierarchical clustering process. Akram et al. [21] delineated trapezoidal PFNs, utilized as network characteristics for assessing maximal flow. Readers are urged to see Akram and Habib [22] for more notations and applications. Habib and Akram [23], Nawaz and Akram [24], and Zahid and Akram [25]. We implement this concept in a specific assignment problem and endeavor to resolve it without transforming it into a Crisps problem. We examine the fuzzy assignment problem, specifically the assignment problem characterized by Pythagorean fuzzy costs. $\widetilde{C_{ij}^{p}}$ in this paper. A model of fuzzy assignment was presented by Chen [12]. Wang [13] used graph theory methods to solve a related model. Numerous ranking techniques can be used to explain the dominance of fuzzy numbers [14]. The Hungarian method was employed by Senthil Kumar et al. [26] to resolve the intuitionistic fuzzy assignment problem. The perfect matching algorithm was implemented by Nagoor Gani et al. [27] to resolve the intuitionistic fuzzy linear bottleneck assignment problem. Dhanasekar et al. [28,29,30,31] solved assignment problem in various fuzzy domain. We should investigate all potential solutions to this issue.

- We produce n! possible job assignments, compute the total cost for each assignment, and identify the most economical assignment. The complexity of the solution is O(n!), as it involves a permutation of n tasks.
- The Hungarian algorithm can be utilized to determine the optimal assignment. The Hungarian method exhibits a worst-case time complexity of $O(n^3)$).
- A state space tree is a N-ary tree characterized by the fact that each path from the root to a leaf node represents a potential solution to a specific problem. We possess the capability to perform a depth-first search on the state space tree; but, following actions may lead to an increased distance from the aim instead of a reduction. The search of the state space tree commences along the leftmost path from the root, irrespective of the initial state. This method may not produce a result node. We may also perform a breadth-first search on the state space tree. Nonetheless, the method performs the same series of operations as Depth-First Search, irrespective of the initial condition.
- The selection criterion for the subsequent node in BFS and DFS is "blind." The selection rule does not favor a node with a high likelihood of swiftly guiding the search to an answer node. The application of a "approximate cost function," or "intelligent" ranking function, sometimes accelerates the search for an optimal solution by circumventing sub-trees that lack an ideal answer. It resembles a search akin to BFS, however with substantial optimization. Instead, than following the FIFO sequence, we choose the active node with the minimal cost. While the best solution may not be achieved by pursuing the node with the least favorable cost, there exists a significant probability of swiftly progressing the search towards an answer node.

The branch and bound method is a reliable technique for addressing optimization problems. The time complication is O(m*n). The Branch and Bound algorithm's primary benefit is its ability to efficiently reduce the search space while simultaneously delivering an optimal solution. Nevertheless, its drawback is that it can devour a significant amount of memory and time for complex problems, contingent upon the characteristics of the search space.In this research, we solve assignment problems with fuzzy costs by applying the branch and bound technique.

This study's primary contributions are as follows:

- Formulation of a triangular Pythagorean fuzzy assignment issue.
- A novel ranking method is proposed to arrange triangular Pythagorean fuzzy numbers.
- The branch and bound approach is utilized for the Pythagorean fuzzy assignment issue.

II. Basics

This part provides fundamental definitions necessary for comprehending the subsequent sections.

2.1 Definition

Let ‘A’ be a classical set a Pythagorean fuzzy set (PFS) [15] is of the form $P = \{(a, \mu_P(a), \vartheta_P(a)) | a \in A\}$ where the function. $\mu_P(a): A \to [0,1]$. $\vartheta_P(a): A \to [0,1]$ are degree of belongingness and non-belongingness of the element $a \in A$ to P respectively, also. $\forall\ a \in A$ it holds that $\mu_P^2 + \vartheta_P^2 \leq 1$. The level of hesitancy is given by $\pi_p(x) = \sqrt{1 - \mu_P^2 - \vartheta_P^2}$. Pictorial representation of PFS is in Figure 1.

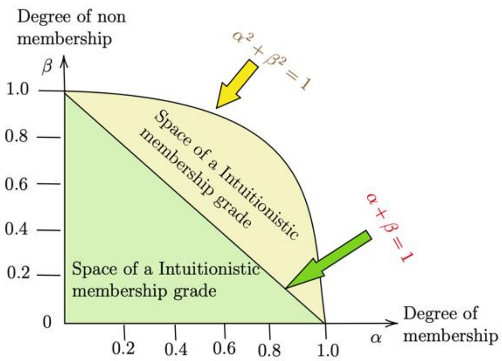


Fig. 1 Belongingness and Non belongingness function

2.2 Defintion

Pythagorean Fuzzy Number [15] can be represented as $a = (\mu_1, \vartheta_1)$ where μ_1 and ϑ_1 membership and non membership function satifies. that $\mu_1^2 + \vartheta_1^2 \leq 1$.

2.3 Definition

A Pythagorean Trapezoidal Fuzzy number $\tilde{P} =< [p_1, p_2, p_3, p_4], m, n >$ [21] is defined by

$$\mu_{\tilde{P}}(a) = \begin{cases} \dfrac{a - p_1}{p_2 - p_1}\, m, & p_1 \leq x \leq p_2 \\ \dfrac{p_4 - a}{p_4 - p_3}\, m, & p_3 \leq x \leq p_4 \\ m, & p_2 \leq x \leq p_3 \\ 0\,, & Otherwise. \end{cases}$$

.

$$\nu_{\tilde{P}}(a) = \begin{cases} \dfrac{p_2 - a + n(x - p_1)}{p_2 - p_1}\,, & p_1 \leq x \leq p_2 \\ \dfrac{a - p_3 + n(p_4 - a)}{p_4 - p_3}\, m, & p_3 \leq x \leq p_4 \\ n, & p_2 \leq x \leq p_3 \\ 1\,, & Otherwise. \end{cases}$$

Where m.n are confidence and nonconfidence levels and indicate the highest and lowest values of $\mu_{\tilde{P}}(x), \upsilon_{\tilde{P}}(x)$ such that $0 \leq m, n \leq 1$ satisfying $m^2 + n^2 \leq 1$.

2.4 Defintion

A Pythagorean Triangular Fuzzy number $\tilde{P} =< [p_1, p_2, p_3], m, n >$ [16] is defined by

$$\mu_{\tilde{P}}(a) = \begin{cases} \dfrac{a - p_1}{p_2 - p_1}\, m, & p_1 \leq x \leq p_2 \\ \dfrac{p_3 - a}{p_3 - p_2}\, m, & p_2 \leq x \leq p_3 \\ 0\,, & Otherwise. \end{cases}$$

$$\nu_{\tilde{P}}(x) = \begin{cases} \dfrac{p_2 - a + n(a - p_1)}{p_2 - p_1}\,, & p_1 \leq x \leq p_2 \\ \dfrac{a - p_2 + n(p_3 - a)}{p_3 - p_2}\, m, & p_2 \leq x \leq p_3 \\ 1\,, & Otherwise. \end{cases}$$

Where m.n are confidence and nonconfidence levels and indicate the highest and lowest values of $\mu_{\tilde{P}}(x), \upsilon_{\tilde{P}}(x)$ such that $0 \leq m, n \leq 1$ satisfying $m^2 + n^2 \leq 1$.

The triangular behavior of the arithmetic operations and the maximum and minimum of the set of triangular and trapezoidal PFNs are guaranteed by the following relationships, which are implemented throughout the work.

2.5 Definition

The arithmetic operations of $\tilde{P} =< [p_1, p_2, p_3], m_1, n_1 >$ and $\tilde{Q} =< [q_1, q_2, q_3,], m_2, n_2 >$ is given by

$\tilde{P} + \tilde{Q} =< [p_1 + q_1, p_2 + q_2, p_3 + q_3], \sqrt{m_1^2 + m_2^2 - m_1^2 m_2^2}, n_1 n_2 >$

$\tilde{P} - \tilde{Q} =< [p_1 - q_3, p_2 - q_2, p_3 - q_1], \sqrt{\frac{m_1^2 - m_2^2}{1 - m_2^2}}, \frac{n_1}{n_2} >$

2.6 Definition

Let $\tilde{P} =< [p_1, p_2, p_3, p_4], m_1, n_1 >$ and $\tilde{Q} =< [q_1, q_2, q_3, q_4], m_2, n_2 >$ be trapezoidal PFNs Then

$\tilde{P} + \tilde{Q} =< [p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4], \sqrt{m_1^2 + m_2^2 - m_1^2 m_2^2}, n_1 n_2 >$

$\tilde{P} - \tilde{Q} =< [p_1 - q_4, p_2 - q_3, p_3 - q_2, p_4 - q_1], \sqrt{\frac{m_1^2 - m_2^2}{1 - m_2^2}}, \frac{n_1}{n_2} >$

2.7 Defintion

Let $\tilde{P} =< [p_1, p_2, p_3], m_1, n_1 >$ and $\tilde{Q} =< [q_1, q_2, q_3], m_2, n_2 >$ be two triangular PFNs. Then the rank of $\tilde{P}$ is defined by

$R(\widetilde{P}) = \left(\frac{p_1 + p_2 + p_3}{3}, m_1^2 - n_1^2, m_1^2 + n_1^2\right) = (\alpha_1, \beta_1, \gamma_1).$

$R(\widetilde{Q}) = \left(\frac{q_1 + q_2 + q_3}{3}, m_2^2 - n_2^2, m_2^2 + n_2^2\right) = (\alpha_2, \beta_2, \gamma_2)..$

- If $\alpha_1 < \alpha_2$ then $\tilde{P} \prec \tilde{Q}$
- If $\alpha_1 > \alpha_2$ then $\tilde{P} \succ \tilde{Q}$
- If $\alpha_1 = \alpha_2$ then
  - I) If $\beta_1 < \beta_2$ then $\tilde{P} \prec \tilde{Q}$
  - II) If $\beta_1 > \beta_2$ then $\tilde{P} \succ \tilde{Q}$
  - III) If $\beta_1 = \beta_2$ then
    - A) If $\gamma_1 < \gamma_2$ then $\tilde{P} \prec \tilde{Q}$
    - B) If $\gamma_1 > \gamma_2$ then $\tilde{P} \succ \tilde{Q}$
    - C) If $\gamma_1 = \gamma_2$ then $\tilde{P} \approx \tilde{Q}$

2.7.1 Example

Let $\tilde{P} = \langle[20,40,60], 1,0\rangle$ and $\tilde{Q} = \langle[20,40,60], 0.7,0.5\rangle$

Then $R(\tilde{P}) = \left(\frac{20+40+60}{3}, 1^2 - 0^2\ 1^2 + 0^2\right) = (40,1,1)$.

$R(\tilde{Q}) = \left(\frac{20+40+60}{3}, 0.7^2 - 0.5^2\ 0.7^2 + 0.5^2\right) =$ (40,0.24,0.74).

By the previous definition $\alpha_1 = \alpha_2, \beta_1 > \beta_2$ therefore $\tilde{P} \succ \tilde{Q}$.

2.8 Formulation of Pythagorean Fuzzy Assignment problem

The cost matrix can be expressed in the form of n×n.

$$\begin{array}{c c} & \begin{array}{cccc} \text{job1} & \text{job2} & . & \text{jobN} \end{array} \\ \begin{array}{c} \text{person1} \\ \text{person2} \\ \text{person3} \\ . \\ \text{personN} \end{array} & \begin{pmatrix} \widetilde{C^p_{11}} & \widetilde{C^p_{12}} & . & . & \widetilde{C^p_{1N}} \\ \widetilde{C^p_{21}} & \widetilde{C^p_{22}} & . & . & \widetilde{C^p_{2N}} \\ \widetilde{C^p_{31}} & \widetilde{C_{32}}^p & . & . & \widetilde{C^p_{3N}} \\ . & . & . & . & . \\ \widetilde{C^p_{N1}} & \widetilde{C^p_{N2}} & . & . & \widetilde{C^p_{NN}} \end{pmatrix} \end{array}$$

In mathematical terms, it can be expressed as Minimize $\tilde{Z} = \sum_{k=1}^{n} \sum_{l=1}^{n} \widetilde{C_{kl}}^P x_{kl}$ subject to

$$\sum_{k=1}^{n} x_{kl} = 1, \sum_{l=1}^{n} x_{kl} = 1, \text{where}$$

$$x_{kl} = \begin{cases} 1 \text{ if } k^{th} \text{ person is assigned to the } l^{th} \text{ job} \\ 0 \quad \text{otherwise} \end{cases}$$

Where $\widetilde{C^p_{11}}$ will be either triangular Pythagorean fuzzy number or trapezoidal Pythagorean fuzzy number. Where $\widetilde{C^p_{11}}$ will be either triangular Pythagorean fuzzy number or trapezoidal Pythagorean fuzzy number.

III. ALGORITHM

3.1 Branch and Bound technique for assignment problem:

Let 'K' denote the level number in the branching tree (for the root node, it is 0). Let σ represent an assignment made in the current node of the branching tree. Let P_σ^K represent an assignment at level K of the branching tree.A is the set of assigned cells (partial assignment) up to the node P_σ^K from the root node (set of I and j values with respect to the assigned cells up to the node P_σ^K from the root node). The lower bound of the partial assignment, A up to P_σ^K, is

$$\widetilde{V^p_\sigma} = \sum_{i,j\epsilon A} \widetilde{C^p_{ij}} + \sum_{I\epsilon x}\left(\sum_{j\epsilon Y} \min \widetilde{C_{ij}}^p\right)$$

Where $\widetilde{C_{ij}}$ is the cell entry of the cost matrix with respect to the i^{th} row and j^{th} column, X is the set of rows that are not deleted up to the node P_σ^K from the root node in the branching tree, and Y is the set of columns that are not deleted up to the node P_σ^K from the root node in the branching tree.

Branching Guidelines:

1. The best column of the assigned problem will be allocated at level K of the row marked as K in the assignment problem.
2. If the lower constraint is tied, the terminal node at the lowest level will be considered for further branching.
3. Stopping Rule: the optimality is achieved if the minimum lower bound is located at any of the terminal nodes at the $(n-1)^{th}$ level. The optimal solution will be formed by the assignments on the path from the root node to that node, as well as the absent pair of row-column combinations

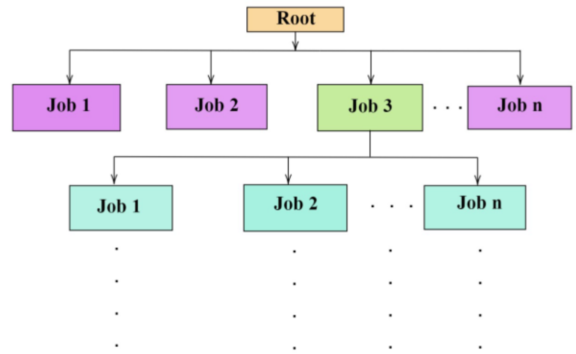


Fig. 2 Diagrammatic Representation of the algorithm.

IV. ALGORITHM

4.1 Example

Let us assume that a project manager is required to assign tasks to its group members in a manner that minimizes the time required to complete the task. Time parameters are unclear in this scenario. To resolve this issue, it has been implemented as Pythagorean triangular fuzzy numbers. The rows in this table represent the four individuals A, B, C, and D, while the columns represent the four occupations Job1, Job2, Job3, and Job4. In Pythagorean triangular fuzzy numbers, the cost matrix is denoted as $(\widetilde{C_{ij}}^p)$. For instance, $\widetilde{C_{11}}^p = < [5,10,15], 1,0 >$ indicates that the time required for person A to complete the task is approximately 10 minutes, with a confidence level of 1 and a pessimistic level of 0

Table 1. Cost Matrix of the 4.1 Example

$$(\widetilde{C_{ij}}^p) = \begin{pmatrix} \langle[5,10,15], 1,0\rangle & \langle[5,10,20], 1,0\rangle & \langle[5,15,20], 1,0\rangle & \langle[5,10,15], 1,0\rangle \\ \langle[5,10,20], 1,0\rangle & \langle[5,15,20], 1,0\rangle & \langle[5,10,15], 1,0\rangle & \langle[10,15,20], 1,0\rangle \\ \langle[5,10,20], 1,0\rangle & \langle[10,15,20], 1,0\rangle & \langle[10,15,20], 1,0\rangle & \langle[5,10,15,1,0\rangle \\ \langle[10,15,25], 1,0\rangle & \langle[5,10,15], 1,0\rangle & \langle[10,20,30], 1,0\rangle & \langle[10,15,25], 1,0\rangle \end{pmatrix}$$

Solution:

A task is not initially assigned to any operator. Therefore, the assignment (σ) at the root node (level 0) of the branching tree is a null set, and the corresponding lower bound $\widetilde{V_\sigma}$ is also 0. So $P_\sigma^0{}^{\sigma=\phi}_{V\phi=0}$

Further branching:

The lower bound for each of the four distinct sub-problems under the root node is provided below:

$P_{11}^1{}^p\langle[20,40,60], 1,0\rangle$, $P_{12}^1{}^p\langle[25,45,75], 1,0\rangle$

$P_{13}^1{}^p\langle[20,45,70], 1,0\rangle$, $P_{14}^1{}^p\langle[20,45,65], 1,0\rangle$

Lower bound for P_{11}^1:

$$\widetilde{V_\sigma}^p = \sum_{i,j\epsilon A} \widetilde{C_{ij}}^p + \sum_{I\epsilon x}\left(\sum_{j\epsilon Y} \min \widetilde{C^p_{ij}}\right)$$

Where σ= {(11)} A= {(11)} X ={2,3,4} Y = {2,3,4}

$\widetilde{V^p_{11}} = \langle 5,10,15\rangle +$

$Min\ \{\langle[5,15,20], 1,0\rangle\langle[10,15,20], 1,0\rangle\langle[5,10,15], 1,0\rangle\}+$

$Min\ \{\langle[10,15,20], 1,0\rangle\langle[10,15,20], 1,0\rangle\langle[5,10,15], 1,0\rangle\}$

+

$Min\ \{\langle[10,15,20],1,0\rangle\langle[10,15,25],1,0\rangle\langle[10,20,30],1,0\rangle\}$

$= \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle = \langle[20,40,60],1,0\rangle .= \langle 20,40,60\rangle$

Further branching:

An additional branch is initiated from the terminal node that has the lowest lower bound. At this stage, the nodes P_{11}^{1}, P_{12}^{1}, P_{13}^{1}, P_{14}^{1} are terminal nodes. Among these nodes the node P_{11}^{1} has least low bound. Consequently, there is additional branching that occurs at this node.

$P_{22}^{2}{}^{p}\langle[25,50,75],1,0\rangle$, $P_{23}^{2}{}^{p}\ \langle[20,40,60],1,0\rangle$

$P_{24}^{2}{}^{p}\ \langle[30,50,70],1,0\rangle$

Lower bound for $P_{23}^{2}{}^{p}$:

$$\widetilde{V_\sigma}^{p} = \sum_{i,j\epsilon A}\widetilde{C_{ij}}^{p} + \sum_{I\epsilon x}(\sum_{j\epsilon Y}\min \widetilde{C_{ij}^{p}})$$

Where σ= {(23)} A= {(23)} X ={3,4} Y = {2,4}

$\widetilde{V_{23}^{p}} = \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle = \langle[20,40,60],1,0\rangle$

Further branching:

At this stage, the nodes P_{11}^{1}, P_{12}^{1}, P_{13}^{1}, P_{14}^{1}, P_{22}^{2}, P_{23}^{2}, P_{24}^{2} are terminal nodes. Among these nodes, the node P_{23}^{2} has the lowest bound. Therefore, this node is the site of additional branching.

$P_{32}^{3}{}^{p}\ \langle[30,50,75],1,0\rangle$, $P_{34}^{3}{}^{p}\ \langle[20,40,60],1,0\rangle$

Lower bound for P_{34}^{3} :

$$\widetilde{V_\sigma}^{p} = \sum_{i,j\epsilon A}\widetilde{C_{ij}}^{p} + \sum_{I\epsilon x}(\sum_{j\epsilon Y}\min \widetilde{C_{ij}^{p}})$$

$\widetilde{V_{34}^{p}} = \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle + \langle[5,10,15],1,0\rangle = \langle[20,40,60],1,0\rangle$

Further branching:

At this stage, the nodes P_{11}^{1}, P_{12}^{1}, P_{13}^{1}, P_{14}^{1}, P_{22}^{2}, P_{23}^{2}, P_{24}^{2}, P_{32}^{3}, P_{34}^{3} are terminal nodes. Among these nodes, the node P_{34}^{3} has the lowest bound. The P_{34}^{3} is at the bottom level. Optimality is achieved because the node is located at the $(n-1)^{th}$ level (K=3) of the branching tree, where n is the number of rows of assignment problem. In addition to the missing pair of person and job combinations, (4,2), the root node is the starting point for the corresponding solution to the node P_{34}^{3}.

Table 2. Optimum Solution of 4.1 Example

person	job	value
A	1	$\langle[5,10,15],1,0\rangle$
B	3	$\langle[5,10,15],1,0\rangle$
C	4	$\langle[5,10,15],1,0\rangle$
D	2	$\langle[5,10,15],1,0\rangle$

The assignment cost is $\langle[20,40,60],1,0\rangle$. The result states that the total assignment time is about 40 mins with a confidence level of 1 and a pessimistic level of 0. The algorithm's diagrammatic representation is illustrated below.

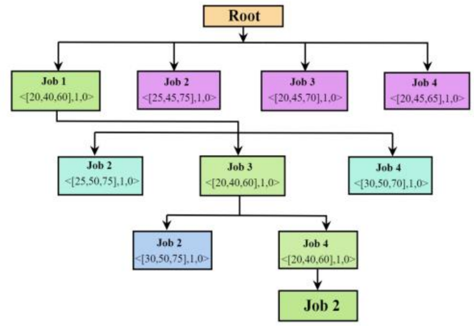


Fig. 3 Schematic representation of the algorithm

4.2 Example

One possibility is to consider a delivery Company manager are required to allocate a set of delivery routes to their fleet of vehicles. The objective is to optimize the allocation of routes to vehicles based on the distance factor. Distance parameters are ambiguous in this scenario. To resolve this matter, Pythagorean triangular fuzzy numbers have been implemented. Rows in this scenario correspond to the five vehicles (A, B, C, D, and E), while columns correspond to the five routes (Route 1, Route 2, Route 3, Route 4, and Route 5). The cost matrix is $\left(\widetilde{C_{IJ}}^{p}\right)$ is given in trapezoidal fuzzy numbers. For instance, $\widetilde{C_{11}}^{p} =< [4,6,7,9],1,0 >$ indicates that the time required for Vechile A to complete the route is about an interval [6,7] minutes, with a confidence level of 1 and a pessimistic level of 0.

Table 3. Cost Matrix of the 4.2 Example

$$(\widetilde{C_{ij}}) = \begin{pmatrix} \langle[4,6,7,9],1,0\rangle & \langle[3,5,7,9],1,0\rangle & \langle[5,7,10,12],1,0\rangle & \langle[3,4,6,9],1,0\rangle & \langle[4,5,7,10],1,0\rangle \\ \langle[2,3,5,9],1,0\rangle & \langle[5,7,9,13],1,0\rangle & \langle[4,6,9,12],1,0\rangle & \langle[5,6,7,10],1,0\rangle & \langle[2,3,5,7],1,0\rangle \\ \langle[7,9,10,12],1,0\rangle & \langle[6,7,9,10],1,0\rangle & \langle[7,9,10,13],1,0\rangle & \langle[6,7,10,13],1,0\rangle & \langle[7,10,13,14],1,0\rangle \\ \langle[4,5,7,9],1,0\rangle & \langle[5,7,12,15],1,0\rangle & \langle[7,9,13,15],1,0\rangle & \langle[2,9,10,13],1,0\rangle & \langle[5,7,10,14],1,0\rangle \\ \langle[4,10,13,15],1,0\rangle & \langle[3,7,9,13],1,0\rangle & \langle[2,3,10,14],1,0\rangle & \langle[3,7,10,13],1,0\rangle & \langle[4,7,10,14],1,0\rangle \end{pmatrix}$$

Solution:

At first, a job is not given to any operator. Because of this, the assignment (π) at the branching tree's root node (level 0) is a null set, and so is the lower bound $\widetilde{V_\sigma}$ So $P_{\sigma}^{0}{}^{\sigma=\phi}_{V\phi=0}$

Further branching:

The lower bound for each of the four distinct sub-problems under the root node is provided below:

$P_{11}^{1}{}^{p}\langle[19,26,41,54],1,0\rangle$, $P_{12}^{1}{}^{p}\langle[17,23,39,52],1,0\rangle$ $P_{13}^{1}{}^{p}\langle[20,29,40,51],1,0\rangle$, $P_{14}^{1}{}^{p}\ \langle[17,22,37,48],1,0\rangle$, $P_{15}^{1}{}^{p}\ \langle[18,23,38,52],1,0\rangle$

Lower bound for P_{14}^{1}:

$$\widetilde{V_\sigma}^{p} = \sum_{i,j\epsilon A}\widetilde{C_{ij}}^{p} + \sum_{I\epsilon x}(\sum_{j\epsilon Y}\min \widetilde{C_{ij}^{p}})$$

Where σ= {(1,4)} A= {(1,4)} X ={2,3,4,5} Y = {1,2,3,5}

$\widetilde{V_{14}}^{p} = \langle 3,4,6,9\rangle$ +

$Min\ \{\langle[2,3,5,9],1,0\rangle, \langle[4,6,9,12],1,0\rangle, \langle[5,7,9,13],1,0\rangle, \langle[2,3,5,7],1,0\rangle\}$

+

$Min\ \{\langle[7,9,10,12],1,0\rangle, \langle[6,7,9,10],1,0\rangle, \langle[7,9,10,13],1,0\rangle, \langle[7,10,13,14]1,0\rangle\}$

+

$Min\ \{\langle[4,5,7,9],1,0\rangle, \langle[5,7,12,15],1,0\rangle, \langle[7,9,13,15],1,0\rangle, \langle[5,7,10,14],1,0\rangle\}$ +

$Min\{\langle[4,10,13,15],1,0\rangle, \langle[3,7,9,13],1,0\rangle, \langle[2,3,10,14],1,0\rangle, \langle[4,7,10,14],1,0\rangle\}$

$= \langle[3,4,6,9],1,0\rangle + \langle[2,3,5,7],1,0\rangle + \langle[6,7,9,10],1,0\rangle + \langle[4,5,7,9],1,0\rangle + \langle[2,3,10,14],1,0\rangle = \langle[17,22,37,49],1,0\rangle$

Further branching:

An additional branch is initiated from the terminal node that has the lowest lower bound. At this stage, the nodes P_{11}^{1}, P_{12}^{1}, P_{13}^{1}, P_{14}^{1} , P_{15}^{1} are terminal nodes. Among these nodes the node P_{14}^{1} has least lower bound. Hence further branching is done from this node. $\widetilde{P_{21}^{2}}^{p}\langle[18,24,40,56],1,0\rangle$, $\widetilde{P_{22}^{2}}^{p}\langle[21,28,42,57],1,0\rangle$, $\widetilde{P_{23}^{2}}^{p}\langle[20,29,40,51],1,0\rangle$ $\widetilde{P_{25}^{2}}^{p}\langle[17,22,37,49],1,0\rangle$

Lower bound for P_{23}^{2}:

$$\widetilde{V_{\sigma}}^{p} = \sum_{i,j\epsilon A}\widetilde{C_{ij}}^{p} + \sum_{I\epsilon x}(\sum_{j\epsilon Y}\min\widetilde{C_{ij}^{p}})$$

Where σ= {(23)} A= {(23)} X ={3,4} Y = {2,4}

$\widetilde{V_{25}}^{p} = \langle[3,4,6,9],1,0\rangle + \langle[2,3,5,7],1,0\rangle + \langle[6,7,9,10],1,0\rangle + \langle[4,5,7,9],1,0\rangle + \langle[2,3,10,14],1,0\rangle$

$= \langle[17,22,37,49],1,0\rangle$

Further branching:

At this stage, the nodes P_{11}^{1},P_{12}^{1},P_{13}^{1}, P_{14}^{1} P_{15}^{1} ,P_{22}^{2}, P_{23}^{2} , P_{25}^{2} P_{21}^{2} are terminal nodes. Among these nodes, the node P_{25}^{2} has the lowest bound. Hence further branching is done from this node. $\widetilde{P_{31}^{3}}^{p}\langle[19,26,43,57],1,0\rangle$,$\widetilde{P_{32}^{3}}^{p}\langle[17,22,37,49],1,0\rangle$ $\widetilde{P_{33}^{3}}^{p}\langle[20,29,40,51],1,0\rangle$

Lower bound for P_{32}^{3} :

$$\widetilde{V_{\sigma}}^{p} = \sum_{i,j\epsilon A}\widetilde{C_{ij}}^{p} + \sum_{I\epsilon x}(\sum_{j\epsilon Y}\min\widetilde{C_{ij}^{p}})$$

$\widetilde{V_{32}}^{p} = \langle[3,4,6,9],1,0\rangle + \langle[2,3,5,7],1,0\rangle + \langle[6,7,9,10],1,0\rangle + \langle[4,5,7,9],1,0\rangle + \langle[2,3,10,14],1,0\rangle = \langle[17,22,37,49],1,0\rangle$

Further branching:

At this stage, the nodes P_{11}^{1}, P_{12}^{1}, P_{13}^{1}, P_{14}^{1} P_{15}^{1},P_{21}^{2} P_{22}^{2}, P_{23}^{2} , P_{25}^{2}, P_{31}^{3}, P_{32}^{3} P_{33}^{3} are terminal nodes. Among these nodes, the node P_{32}^{3} has the lowest bound. Hence further branching is done from this node $P_{41}^{4}\langle[17,22,37,49],1,0\rangle$, $P_{43}^{4}\langle[22,33,46,56],1,0\rangle$.

Lower bound for P_{41}^{4} :

$$\widetilde{V_{\sigma}}^{p} = \sum_{i,j\epsilon A}\widetilde{C_{ij}}^{p} + \sum_{I\epsilon x}(\sum_{j\epsilon Y}\min\widetilde{C_{ij}^{p}})$$

$\widetilde{V_{41}}^{p} = \langle[3,4,6,9],1,0\rangle + \langle[2,3,5,7],1,0\rangle + \langle[6,7,9,10],1,0\rangle + \langle[6,7,9,10],1,0\rangle + \langle[4,5,7,9],1,0\rangle + \langle[2,3,10,14],1,0\rangle = \langle[17,22,37,49],1,0\rangle$

Further branching:

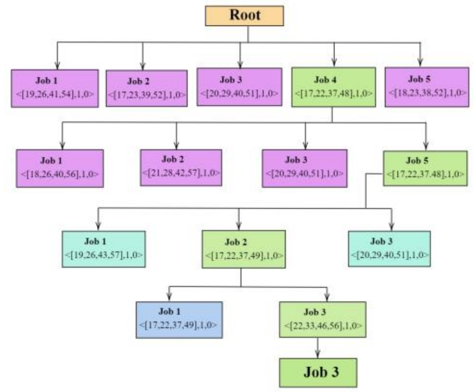
At this stage, the nodes P_{11}^{1}, P_{12}^{1}, P_{13}^{1}, P_{14}^{1} P_{15}^{1},P_{21}^{2} P_{22}^{2}, P_{23}^{2} , P_{25}^{2}, P_{31}^{3}, P_{32}^{3} P_{33}^{3} P_{41}^{4} P_{43}^{4} are terminal nodes. Among these nodes, the node P_{41}^{4} has the lowest bound.

Optimality is achieved because the node is located at the $(n-1)^{th}$ level (K=4) of the branching tree, where n is the size of the assignment problem. In addition to the missing pair of person and job combinations, (5,3), the corresponding solution is traced from the root node to the node P_{41}^{4}.

Table 4. Optimum solution of the 4.2 Example

person	job	value
A	4	$\langle[3,4,6,9],1,0\rangle$
B	5	$\langle[2,3,5,7],1,0\rangle$
C	2	$\langle[6,7,9,10],1,0\rangle$
D	1	$\langle[4,5,7,9],1,0\rangle$
E	3	$\langle[2,3,10,14],1,0\rangle$

The assignment cost is $\langle[17,22,37,49],1,0\rangle$. The result states that the total time to deliver is about [22,37] mins with a confidence level of 1 and a pessimistic level of 0. The diagrammatic representation of the algorithm is given below



V. CONCLUSION

The approach that has been demonstrated is an exceedingly beneficial instrument for the integration of PF parameters into mathematical models in order to mitigate uncertainties. The current research has shown that the integration of PFNs as measures for both time and cost in the evaluation of activities can lead to a more realistic extension. Therefore, it is recommended that an effective approach be employed to determine the optimal solution to the assignment problem. In order to resolve the ambiguous assignment issue, a new algorithm is proposed. This algorithm is both effective and straightforward to understand due to its resemblance to the traditional branch and bound technique. Defuzzification is not necessary for this technique to be effective. This technique is more effective than the current method because the results are presented in imprecise numbers. This method is effective in resolving assignments that are unbalanced. This work has the potential to be further developed into:

- Pythagorean Fuzzy Transportation Modeling.
- Pythagorean Fuzzy Traveling Salesman Modeling.
- The simulation of Fermi-an optimization problems.

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